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Letter to the Editor

Multi-stage regression analysis of acoustical properties of polyurethane foams

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1. Introduction

A quantitative measure of the acoustic energy absorption efficiency of sound absorbing materials is often needed in designing noise control treatments. In this work, we consider rigidly backed and locally reacting polyurethane foams, for which the normal incidence absorption coefficient α_n is such a measure. This letter presents a multi-stage regression modelling approach for the prediction of α_n of this class of foams.

The absorption coefficient α_n can be measured by some standard test procedures [1]. There have been many studies attempting to predict it from material properties including airflow resistivity, porosity, elastic constants, pore geometry, and so on. Such a relation if established can provide useful tools for designing acoustic treatments. The approaches commonly taken include empirical regression models, and theoretical models based on phenomenological or microstructural material parameters [2–4]. The material parameters for the microstructural models need to be determined from a detailed description of the material microstructure, ultrasonic experimentation [5], or other experimental techniques. The phenomenological models employ frequency-dependent parameters such as the structure factor and the non-adiabatic bulk modulus to modify the linear acoustic equations of force and continuity [6–11]. Phenomenological models do not specify how these parameters might be measured or calculated.

Delaney and Bazley [10,12] used regressions to find empirical power-law relationships between the non-dimensional parameter $\rho\omega/(2\pi\sigma)$, the characteristic impedance Z_c and the propagation constant Γ for fibrous sound absorbing materials. Similar models have been developed for plastic foams [13]. Bies and Hansen [14] used the data reported in Refs. [10,12] to develop a polynomial model. Note that the characteristic impedance Z_c and the propagation constant Γ are related to the normal incidence absorption coefficient α_n. An example of this relationship for one-dimensional ducts is given in Ref. [6]. A neural network model was developed in Ref. [15] to predict α_n and Z_c with frequency and flow resistivity as input. The present work is an extension of

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the work in Ref. [15], and develops a multi-stage regression modelling approach for the same set of acoustic data used in this reference. Some examples of multi-stage regression can be found in Refs. [16,17].

The remainder of the paper is organized as follows. In Section 2 we briefly introduce the experimental set-up to measure the normal incidence absorption coefficient. Some typical experimental data are presented. In Section 3, a multi-stage regression method is proposed to obtain empirical formula of the normal incidence absorption coefficient as a function of airflow resistivity, frequency, foam thickness and mass density. An iterative procedure is constructed to improve the accuracy of the overall prediction of α_n. Section 4 evaluates the effectiveness of the predictions of the empirical formula with the experimental data not used in the regression analysis. Some conclusions are drawn in Section 5.

2. Experimental set-up and results

An experimental test set-up is built according to an ASTM standard [1] for measuring the normal incidence absorption coefficient α_n of the polyurethane foams. The schematic of the experimental set-up is shown in Fig. 1, which is designed to be effective in the frequency range from 200 to 2000 Hz.

The theory underlying the experimental set-up is known as two microphone random excitation method. Details of the theory can be found in Ref. [18]. The complex pressure reflection coefficient R at the surface of the foam in the test tube can be expressed as

$$R = \frac{(S_{12}/S_{11}) - \exp(-\mathrm{j}ks)}{\exp(\mathrm{j}ks) - (S_{12}/S_{11})} \exp(2\mathrm{j}kl), \tag{1}$$

where s is the distance between the two microphones, k is the acoustic wavenumber in the air at a given frequency f, l is the distance from the first microphone to the surface of the foam, and S_{12}/S_{11} is the ratio of the cross and auto-spectral density functions between the two microphones.

The normal incidence absorption coefficient is given in terms of R as

$$\alpha_n = 1 - |R|^2. \tag{2}$$

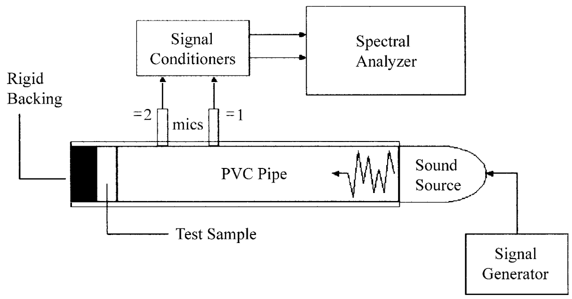

Fig. 1. Schematic of the experimental set-up for testing the normal incidence absorption coefficient.

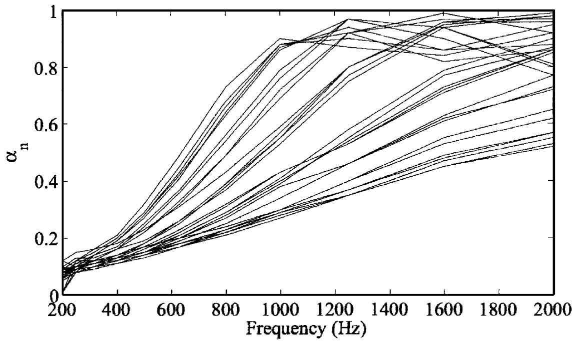

Fig. 2. Experimental data of normal incidence absorption coefficient vs. frequency for 29 polyurethane foams. Each curve corresponds to a sample with a known airflow resistivity.

Fig. 2 shows the absorption curves of 29 partially reticulated polyurethane foams studied in Ref. [15]. Each curve corresponds to a material with a known airflow resistivity, mass density and thickness.

3. Regression analysis of α_n

As discussed above, the normal incidence absorption coefficient α_n is generally a function of airflow resistivity, frequency, thickness and mass density. Regression analysis of α_n is therefore a multi-dimensional curve-fitting problem, which requires a large set of experimental data due to the so-called curse of dimensionality in statistics. To obtain an extensive set of experimental data in order to numerically model α_n is a prohibitive task. Moreover, it is difficult to choose a function form for α_n, which is a critical step in regression analysis. With a limited but reasonable set of experimental data, we attempt to develop a multi-stage regression approach to create a functional relationship between α_n and the other material parameters.

Regression is a well-developed method, and is documented in many textbooks [19,20]. We shall skip the introduction of the method herein.

In the first stage, we identify a function form in the frequency domain for α_n valid for all the materials shown in Fig. 2, and establish a regression model linking α_n to the frequency. In the second stage, we collect the coefficients from the regression analysis for all the materials, and develop a regression model for each of the coefficients as a function of a compound material parameter consisting of airflow resistivity, thickness and mass density. The result of the multi-stage regression modelling is an empirical formula for predicting the absorption coefficient α_n.

3.1. Measure of goodness of fit

Assume that there exists a function relating α_n to the parameters of interest as follows:

$$\alpha_n = F(f, \sigma, \rho, T), \tag{3}$$

where f is the frequency, σ is the airflow resistivity with unit Pa $\mathrm{s/m^2}$, ρ is the foam mass density, and T is the thickness of the sample. The regression amounts to using a set of experimental data to estimate the function $F(\cdot)$.

Let $\hat{F}(\cdot)$ denote the function chosen in the regression analysis to approximate $F(\cdot)$. The sum of the squared deviations between the observed data α_{ni} and the prediction of the model for a given material is given by

$$e_k^2 = \sum_{i=1}^{N_k} [\alpha_{ni} - \hat{F}(f_i, \sigma_i, \rho_i, T_i, \mathbf{a})]^2, \tag{4}$$

where k is an index referring to the material under consideration, $\mathbf{a}$ denotes a set of parameters to be determined so that e_k^2 is minimized, and N_k is the number of data points of the material.

To examine the accuracy of global prediction of the regression among all the materials, a measure of goodness of fit is adopted here as defined below:

$$e_{fit} = \sum_{k=1}^{M} \frac{\sqrt{(1/N_k)e_k^2}}{\sum_{i=1}^{N_k} \alpha_{ni}}, \tag{5}$$

where M is the total number of materials considered in the regression.

3.2. The first stage

Fig. 2 clearly shows that the absorption coefficient is a function of frequency. After several trials, we have come to the following function form:

$$\alpha_n = \frac{a_1}{1 + \exp[a_2(1 - a_3 \ln f)]}, \tag{6}$$

where a_i $(i = 1, 2, 3)$ are unknown parameters, which assume different values for different foams. Fig. 3 shows the coefficients as a function of a compound material parameter $\sigma T/\rho$. The compound parameter $\sigma T/\rho$ often appears in analytical models of polymeric foams.

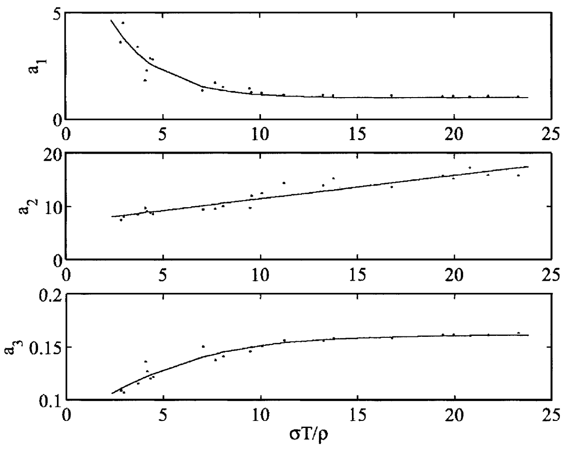

Fig. 3. The coefficients of the regression function for the absorption coefficient and the regression of these coefficients at the second stage. Dot: the coefficients from the first stage regression. Solid line: fitting curves of these coefficients.

We will fit these parameters as functions of the compound parameter in the second regression stage. Note that f is the first stage predictor, and the foam thickness, density and airflow resistivity become the candidates of the second stage predictors. a_i are the second stage response variables.

3.3. The second stage

In the second stage of regression, there are three functions of the compound parameter $\sigma T/\rho$ to be determined for a_i, respectively. After several trials, we obtain the following results:

$$a_1 = 1 + \exp\left(2.25128 - 0.40904\frac{\sigma T}{\rho}\right), \tag{7a}$$

$$a_2 = 0.43727\frac{\sigma T}{\rho} + 7.1249, \tag{7b}$$

$$a_3 = \frac{1}{6.18572 + \exp(1.78624 - 0.26442(\sigma T/\rho))}. \tag{7c}$$

3.3.1. A remark

A remark is in order on the second stage regression. Clearly, $a_i\,(i = 1, 2, 3)$ are in general functions of airflow resistivity σ, mass density ρ, and thickness of the sample T. By using the compound parameter $\sigma T/\rho$, we effectively reduce the three-dimensional regression problem for determining the coefficients a_i to several one-dimensional regression problems, and consequently alleviate the need for a large number of data. This is the essence of the present method. Fig. 3 shows the regression results of a_i.

The final expression for α_n becomes

$$\alpha_n = \frac{1 + \exp(2.25128 - 0.40904(\sigma T/\rho))}{1 + \exp\left[(0.43727(\sigma T/\rho) + 7.1249)\left(1 - \frac{\ln f}{6.18572 + \exp(1.78624 - 0.26442(\sigma T/\rho))}\right)\right]}. \tag{8}$$

The measure of goodness of fit of the regression result in Eq. (8) is calculated to be 0.216 over 29 materials.

Note that the regression result in the form of empirical formula (8) can be used by engineers in designing acoustic treatments.

3.4. An iterative procedure

One way to improve the accuracy of the overall prediction of α_n is to introduce an iterative procedure to improve the fitting of the parameters $a_i\,(i = 1, 2, 3)$.

In the iterative regression, function (7c) for a_3 is substituted into Eq. (6) first, and a_1 and a_2 are treated as unknown parameters. The updated regression model now becomes

$$\alpha_n = \frac{a_1}{1 + \exp\left[a_2\left(1 - \frac{\ln f}{6.18572 + \exp(1.78624 - 0.26442(\sigma T/\rho))}\right)\right]}. \tag{9}$$

The choice of the function for a_3 to be substituted first is not crucial. The first and second stage regressions are repeated. A new function for a_1 is found to be

$$a_1 = 1 + \exp\left(1.82193 - 0.315502\frac{\sigma T}{\rho}\right). \tag{10}$$

Note that the coefficients in Eq. (10) are different from those in Eq. (7a). Eq. (10) is substituted in Eq. (9) to update the regression model leaving a_2 as an undetermined parameter. The updated regression model with only one parameter now reads

$$\alpha_n = \frac{1 + \exp(1.82193 - 0.315502(\sigma T/\rho))}{1 + \exp\left[a_2\left(1 - \frac{\ln f}{6.18572 + \exp(1.78624 - 0.26442(\sigma T/\rho))}\right)\right]}. \tag{11}$$

After following through the two stages of regression, we obtain a new function for a_2,

$$a_2 = 0.60125\frac{\sigma T}{\rho} + 5.95842. \tag{12}$$

Note again the difference of the coefficients in Eqs. (12) and (7b). At this point, a new regression function for α_n is obtained,

$$\alpha_n = \frac{1 + \exp(1.82193 - 0.315502(\sigma T/\rho))}{1 + \exp\left[(0.60125(\sigma T/\rho) + 5.95842)\left(1 - \frac{\ln f}{6.18572 + \exp(1.78624 - 0.26442(\sigma T/\rho))}\right)\right]}. \tag{13}$$

The measure of goodness of fit of Eq. (13) is calculated to be 0.208 over the same 29 materials, representing a 3.7% percent improvement over Eq. (8).

In the second iteration, we keep the newest function forms for a_1 and a_2 in Eq. (13) while treating a_3 as an unknown parameter, and then treating a_1 and a_2 as unknowns one at a time subsequently. After updating all three a_i, another new function of α_n is obtained as

$$\alpha_n = \frac{1 + \exp(1.78076 - 0.2982(\sigma T/\rho))}{1 + \exp\left[(0.55309(\sigma T/\rho) + 6.10835)\left(1 - \frac{\ln f}{6.09477 + \exp(1.72744 - 0.234268(\sigma T/\rho))}\right)\right]}. \tag{14}$$

The measure of goodness of fit of Eq. (14) is calculated to be 0.202 over the same 29 materials, representing a 6.5% percent improvement over Eq. (8) and a 2.9% percent improvement over Eq. (13).

This iteration process can continue until the changes in functions for a_i as well as the prediction of α_n become negligible. While it is difficult to analytically prove the convergence of the iterative procedure, the anecdote evidence such as the measure of goodness of fit and the predictions shown in Fig. 4 do support the assertion. Fig. 4 shows the predictions of α_n for several typical foams by using the regression results in Eqs. (8), (13) and (14). The improvement in the prediction due to iterations is visible in the figure.

4. Validation of the model

In order to test the prediction capability of the regression results, new experimental data of α_n of four additional foams are generated and used to check the accuracy of the prediction. The

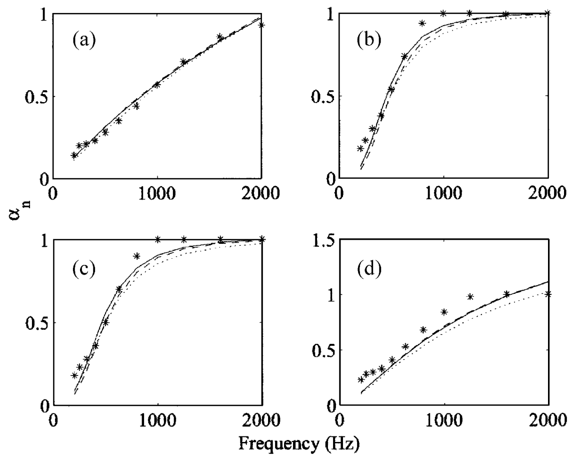

Fig. 4. Comparison of the prediction of the normal incidence sound absorption coefficients of four typical materials. *: the original experimental data. Dotted line: the prediction by Eq. (8). Dashed line: the prediction by Eq. (13). Solid line: the prediction by Eq. (14). The four materials are as follows. (a) $\rho = 30.2909\ \mathrm{kg/m^3}$, $T = 0.02451$ m, $\sigma = 5540\ \mathrm{Pa\ s/m^2}$. (b) $\rho = 28.2568\ \mathrm{kg/m^3}$, $T = 0.02438$ m, $\sigma = 27000\ \mathrm{Pa\ s/m^2}$. (c) $\rho = 28.0415\ \mathrm{kg/m^3}$, $T = 0.02464$ m, $\sigma = 22700\ \mathrm{Pa\ s/m^2}$. (d) $\rho = 28.1527\ \mathrm{kg/m^3}$, $T = 0.02502$ m, $\sigma = 7930\ \mathrm{Pa\ s/m^2}$.

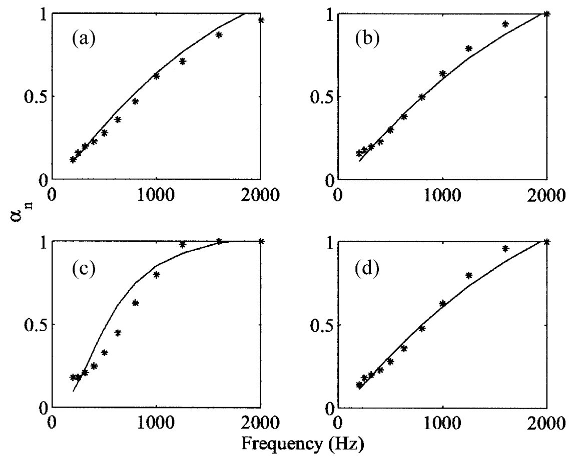

Fig. 5. Validation of the regression model of the normal incidence absorption coefficient of the four additional materials. *: the original experimental data. Solid line: the prediction by Eq. (14). The materials and the fitting errors are as follows. (a) $\rho = 27.6739\ \mathrm{kg/m^3}$, $T = 0.02479$ m, $\sigma = 6920\ \mathrm{Pa\ s/m^2}$, $e_{fit} = 0.00918$. (b) $\rho = 29.2104\ \mathrm{kg/m^3}$, $T = 0.02479$ m, $\sigma = 6520\ \mathrm{Pa\ s/m^2}$, $e_{fit} = 0.00634$. (c) $\rho = 27.2819\ \mathrm{kg/m^3}$, $T = 0.02489$ m, $\sigma = 15,100\ \mathrm{Pa\ s/m^2}$, $e_{fit} = 0.0149$. (d) $\rho = 27.8333\ \mathrm{kg/m^3}$, $T = 0.02515$ m, $\sigma = 6240\ \mathrm{Pa\ s/m^2}$, $e_{fit} = 0.00722$. Note that the measure of goodness of fit e_{fit} presented here is for the material under consideration only.

comparison of the experimental data and the regression prediction from Eq. (14) is shown in Fig. 5. It can be seen that the measures of goodness of fit, e_{fit}, between the experimental data and the regression predictions are less than 2%. The prediction using Eq. (14) agrees well with the experimental data. This supports the regression prediction beyond the set of materials with which the regression model is created.

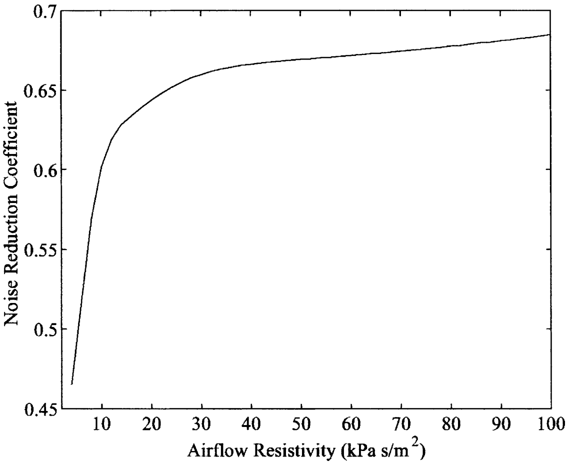

Fig. 6. Noise reduction coefficient vs. airflow resistivity as predicted by Eq. (14).

4.1. An application note

To demonstrate the utility of the regression model in designing acoustic treatments, we present an application note. The noise reduction coefficient (NRC), defined as the arithmetic mean of α_n at 250, 500, 1000 and 2000 Hz, is a parameter commonly used to characterize sound absorbing materials. Fig. 6 shows the predicted NRC as a function of airflow resistivity. We can see that for 1-in thick polyurethane foams with mass density $\rho = 30.2909\ \mathrm{kg/m^3}$, the NRC value achieves significant level with airflow resistivity above 20 $\mathrm{kPa\ s/m^2}$. This is in agreement with the finding in Ref. [15]. To meet the noise reduction requirement as specified by NRC, Fig. 6 suggests a proper range of the airflow resistivity within which the engineer can select a foam.

Many noise sources, such as fans, gears, saws, and internal combustion engines, radiate sound at one or more discrete frequencies. The empirical formula developed here can be used to generate plots similar to Fig. 6, which will aid noise control engineers in selecting a proper partially reticulated polyurethane foam with a right airflow resistivity value for their particular applications.

5. Conclusions

A multi-stage regression method for modelling experimental data of the normal incidence absorption coefficient of polyurethane foam has been developed in this paper. The multi-stage regression converts a difficult high dimensional regression problem into several low-dimensional regression problems. An iterative procedure has also been proposed to improve the accuracy of the regression analysis. In the end, the multi-stage regression analysis leads to an explicit empirical equation relating the normal incidence absorption coefficient to the frequency, mass density, foam thickness and airflow resistivity. Additional experimental data have been used to validate the regression prediction. The agreement between the prediction by the regression and the experimental data is good.

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